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## A Simulation Based Approach to Model Design Influence on the Fatigue life of a Vented Brake disc

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### Abstract

The brake disc is considered a safety critical components in vehicles, hence the growing concern on its service life performance. Brake disc performance is measured by several criteria of which prominent amongst these criteria is fatigue life and disc thermal deflection. This study considers the influence of geometric design features of a vented brake disc on its fatigue life at particular sections of the brake disc which are considered critical and its deflection due to thermal inputs. A parametric study is carried out with CAE/FEA using Taguchi design of experiment. The study identified the geometric design features that significantly influence the studied performance measures. Sensitivity plots were also obtained to show the manner these design factors affect the fatigue life at these points as well as the disc thermal deflection. Two design features, the inboard plate thickness and the length of the effective offset are observed to contribute majorly to the fatigue life of the brake disc as well as its thermal deflection. Hence, design effort should be concentrated on these features for optimal fatigue life design at these points of interest in this study.

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**Keywords:** Degradation; Fatigue; Finite element; Service life; Taguchi

### 1. Introduction

Mechanical damage of structural materials of machine components are generally attributable to factors such as load, temperature, corrosion, time and their interactions, which in connection with component design features, manufacturing process and mechanical properties can intensify the damage [1]. One of such predominant damage mechanisms is fatigue [2]. According to Stephen et al. [3] between seventy and ninety percent of mechanical damage of structures are as a result of fatigue during the course of their operations. One such component prone to fatigue damage is the brake disc. Fatigue in the form of thermal fatigue is a problem of the brake disc as a result of its being subjected to alternating thermal loads (heating and cooling), and constrained in a manner that restricts its free contraction and expansion

[4]. Studies have indicated that damage of the brake disc as a result of thermal cracking is a low cycle thermal-mechanical fatigue [5], and that critical design load cases for the brake discs are most often related to the thermal load [6]. Thus, for a proper investigation of the life of the brake disc a study of thermal effects on the brake disc has to be done.

### Nomenclature

b	Fatigue strength exponent
c	Fatigue ductility exponent
E	Elastic modulus, MPa
$\sigma_y$	Ultimate tensile stress, MPa
$\epsilon_t$	Fatigue ductility coefficient

Researchers have studied the thermal effects on brake disc performance and life using methods such as experiments [4], empirical analysis [5] and with numerical methods [7,8]. In the use of these methods, finite element modelling, a numerical method has found consistent usage in the study of brake discs. In this regard, Belhocine and Bouchetara [8] using a numerical method, finite element analysis modelled the temperature distribution in a disc brake to identify the factors and the entering parameters associated with the time of braking such as the braking mode, geometric design and the brake material. Tirovic [9] also applying finite element analysis identified the disc cooling to aerodynamic efficiency ratio as a useful parameter to assess in developing new brake design or the comparing of different railway brake disc designs. Using both experiments and finite element modelling [10] studied the effect of hole layout on crack initiation in the brake disc. Okamura and Yumoto [11] carried out a series of Computer Aided Engineering/Finite Element Analysis (CAE/FEM) experiments using Taguchi method to demonstrate the effect of basic configurations of brake discs on their thermal behaviour. Their results showed good correlation with experimental data. Duzgun [12] using finite element modelling studied the effect of ventilated disc configurations on thermo-mechanical behaviour of these discs. Based on their study of three different configurations of ventilated disc, they came to the conclusion that the design of the heat dissipation surface of the brake disc has a significant influence on the thermal stress behaviour of these brakes. These researches show the benefit of using finite element modelling for the thermal analysis as well as development of the brake disc. The use of FEM can lead to considerable savings in time and money as it does not involve physical experiments that can be costly and time consuming.

The influence of design features on mechanical components has not received much attention as most studies in component degradation are mostly limited to degradation of material used against the operating or environmental conditions thus leading to the need for increased understanding of design influence on the service life of products [13]. In this research, a study of the influence of geometric design parameters on the thermal fatigue life of a vented brake discs is presented as well as the methodology for carrying out the study. Previous studies have focused on the temperature and thermal stress behaviour of the brake disc but not on the influence of design features on brake disc thermal fatigue life. A design of experiment approach using Computer Assisted Engineering (CAE) and Finite element analysis (FEA) was utilized. Taguchi method of experimental design was used to determine the relative significance of geometric design parameters on the fatigue life of the brake disc at critical sections of the disc, and the disc thermal deflection which also has an influence on brake disc service life, as well as the sensitivity of the thermal fatigue life and deflection of the brake disc to these parameters.

## 2. Methodology

To achieve the purpose of the work which is a study of the influence of geometric design features on brake disc thermal

life, an integrated CAE and a design of experiment approach incorporating finite element analysis was used. The research method involved a simulation of the thermal stresses on the brake discs as result of brake application from which the fatigue life is then determined. The thermal stress and fatigue life of the brake discs are determined through use of finite element analysis a numerical method. The procedure for carrying out the methodology is grouped into several stages. A developed FE model incorporating the brake disc geometry is developed at the initial stage. Next a thermal analysis is performed on the brake disc model to determine the temperature and thermal stresses as a result of brake application. The thermal fatigue life is determined at the third stage using the temperature and thermal stresses obtained in the second stage as inputs. The fourth stage involves a parametric study of the brake discs geometric design parameters using the Taguchi method which is a design of experiment method. Based on the design of experiment matrix obtained the relative influence of the design parameters on the chosen life performance measures of the brake disc is determined as well as parameter sensitivity.

### 2.1 Taguchi method

Taguchi method which has found wide application is a design of experiment method that is used for minimizing product performance variation, and for getting the performance characteristic as close as possible to the targeted mean. Taguchi method is based on orthogonal array (OA) experimental matrix. The use of OA causes a reduction in the variance for the experimental runs resulting in optimum setting of the product/process parameters. Coupled with the use of OA, Taguchi proposed the analysis of product variation using an appropriately selected measure called Signal-to-Noise ratio (SN ratio) which is derived from the quality loss function [14] and can be used as the objective function for optimisation purposes. An advantage of the SN ratio is that it can reflect the variability in the response, and does not induce unnecessary complications such as control factor interactions. The use of Taguchi analysis of the SNR involves three kinds of quality characteristics; smaller the better, larger the better, and nominal the better.

Larger the better,

$$SN\ Ratio = -10\log_{10} \left[ \frac{1}{n} \sum_{i=1}^n (1/Y_i^2) \right] \quad (1)$$

Smaller the better,

$$SN\ Ratio = -10\log_{10} \left[ \frac{1}{n} \sum_{i=1}^n (Y_i^2) \right] \quad (2)$$

Nominal the best,

$$SN\ Ratio = -10\log_{10}(S_i^2/Y_i^2) \quad (3)$$

Where  $n$  is the number of experiments and  $Y_i$  the measured  $i$ th quality, which is response indicator.

## 3. The case study

This case study is concerned with the identification of the geometric design features that significantly influence certain

life performance measures of the brake disc. In this study there are eleven control factors which are the design features and they are to be varied at three different levels. The design parameters and life performance measures were selected based on previous work by [14]. In this study the selected life performance characteristics (responses) of the brake disc are the thermal fatigue life at selected critical points and brake disc deflection due to thermal inputs. Thermal deflection as used in this study refers to the axial movement of the friction plate as a whole. The critical points selected for the thermal fatigue determination are the hat-friction plate corner and hub mounting surface corner.

For this study a Computer Assisted Engineering/Finite Element Analysis (CAE/FEA) with a design of experiment method is used to simulate the thermal behaviour of a vented brake disc for the evaluation of the responses. A series of simulation experiments is carried out using a Taguchi design of experiments to identify the design features that significantly affect the life performance measures. A schematic diagram of the vented brake disc showing the design features is as shown in Fig. 1 while Table 1 gives the design features at their respective levels. The interaction amongst the design features (control factors) is considered negligible. Based on the number of design features and their levels a  $L_{27}$  Taguchi orthogonal array is selected to proceed with the FEA computer experiments

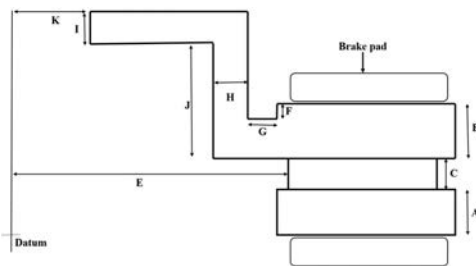


Figure 1. Brake disc diagram showing design features

Table 1. Design factors and their levels

Designation	Parameter	Level 1 (mm)	Level 2 (mm)	Level 3 (mm)
A	Inboard plate	5	7	9
B	Outboard plate	5	7	9
C	Vane height	4	6	8
D	Vane thickness	3	4	5
E	Vane inner radius	78.5	80.5	82.5
F	Undercut depth	1	2	3
G	Undercut thickness	2	4	6
H	Hat-wall thickness	4	6	8
I	Hat thickness	4	6	8
J	Effective Offset	15	25	40
K	Center hole radius	25	27.5	30

### 3.1. FEA model description

A sequentially coupled thermal and structural model of a vented brake disc for a single stop braking mode is constructed using Finite element method. This method involves the application of nodal temperatures obtained from a thermal analysis to a subsequent structural stress analysis as body loads to obtain the stresses and strains using Abaqus FE software. The stress and strain results obtained from the thermal-structural analysis are then used as inputs for the fatigue life determination. The fatigue life is determined using the software Fe-Safe, which is a third party software for Abaqus FE software which has not a solver for the fatigue life. The material of interest being gray cast iron was modelled using the Smith Watson Topper (SWT) model for fatigue life determination. The fatigue life modelled here refers to the time to crack initiation. Linear FEA was used and the elastic stress estimates corrected at the critical points into elastic-plastic stress and strains using Neuber's correction method.

The model is an eight degrees three dimensional symmetrical segment of a vented brake disc. To simulate the single stop braking a time varying uniform heat flux is applied on the friction surfaces of the brake rotor. A uniform heat flux is applied based on the assumption that due to the high rotational speed of the disc, the contact the brake pads make with the disc can be considered as a ring contact and so the heat input on the disc rotor can be assumed to be uniform. In simulating the complete single stop at braking cooling period of the disc is not included in the analysis as this has no significant influence on the thermal behaviour of a brake disc for a single stop braking mode. A braking characteristics of a vehicle traveling initially at 28m/s and brought to a stop within four seconds of applying the brakes is used. The vehicle braking data is as presented in Table 2.

Table 2. Braking characteristics

Parameter	Value
Disc outer diameter, $D_o$ (mm)	255
Disc inner diameter, $D_i$ (mm)	155
Deceleration, $a$ ( $m/s^2$ )	7
Correction factor for rotating mass, $K$	1.1
Heat proportion transferred to disc, $K_p$	0.95
Area of heated disc surface, $A$ ( $m^2$ )	0.032
Braking force fraction of front wheel, $x_f$	0.60
Mass of vehicle, $M$ (kg)	1500
Number of braking pads front axle, $n$	4

The heat flux is determined using the following relationship by [15].

$$Q = \frac{K \times x_f \times K_p \times V \times a \times M}{n \times A} \quad (4)$$

Where  $Q$  is the heat flux density and for this paper it was calculated to be  $1440140.63 \text{ W/m}^2$ . The brake disc under consideration is made of grey cast iron with a density of

7100kg/m<sup>3</sup>, Poisson's ratio is 0.26, Young's modulus of 114000 MPa, a conductivity of 53.3 W/m K, a specific heat capacity of 430 J/kg K, and a thermal expansivity of  $1.1 \times 10^{-6}$ /K [16]. Aside from the material properties of the brake disc other parameters such as the residual stress and surface roughness due to manufacturing were obtained through tests of the sample brake disc used in this study respectively as 34MPa and 0.24 $\mu$ m. These parameters were included in the analysis to improve the prediction ability of the FEA simulation as literature has shown that these influence fatigue life [17]. Table 3 shows the values for the parameters for estimating fatigue life in this study.

Table 3. Fatigue life estimation parameters

E (MPa)	$\sigma_y$ (MPa)	$\epsilon_f$	b	c
114	214	0.007	-0.1176	-0.3011

#### 4. SN Ratio and sensitivity analysis

Taguchi method was used to convert the obtained responses from the CAE/FEA to the SN ratios. The significant factors are determined through graphing of the effects and an analysis of variance (ANOVA) of the SN ratios [14]. The performance measures obtained at each experimental run is converted to SN ratios based on the quality requirement of that performance measure. A larger the better type for Fatigue at hat-friction plate corner and Fatigue at hub mounting surface corner is selected as the aim would be to design for the best fatigue life. For the thermal deflection a smaller the better is selected as the desirable quality index for the reason that the deflection impacts negatively on the service life of the brake disc. The CAE/FEA experimental results are listed in Table 4. All the statistical analysis were performed with Minitab software.

To determine which of the design features have significant influence on the composite performance measure an analysis of variance ANOVA is performed on the obtained SN ratios of the performance measures. The ANOVA of the SN ratios was carried out at a confidence level of 95% with the criteria that any design feature parameter with a P-value less than 0.05 are considered significant. The ANOVA results displayed in Tables 5, 6 and 7 give the influential parameters for the responses. The influential parameters for fatigue at the hat-friction plate corner are the inboard plate, outboard plate, undercut depth, undercut thickness and the effective offset. While for fatigue at hub corner the influential parameters are the inboard friction plate, hat thickness, effective offset, and the hat-wall thickness. The ANOVA for the axial deflection of the discs shows that the significant parameters that influence it majorly are the inboard plate, outboard plate, effective offset and the vane height.

Observing the sensitivity plots for the responses a trend can be observed concerning certain parameters. It can be observed that the inboard plate and effective offset occur as significant parameters in the three performance criteria. Figure 2 shows the plot of the factorial effects on fatigue life at the corner between the hat and the friction plate, Figure 3, the sensitivity plot for the Fatigue at the Hub corner mounting surface corner, and likewise Figure 4 the sensitivity plot for the thermal deflection.

Table 4. CAE/FEA Experimental data for performance measures

Run	Axial deflection (mm)	Fatigue life at Hat-friction plate corner (Reversals to failure)	Fatigue life at Hub mounting surface corner (Reversals to failure)
1	0.3063	39810	79
2	0.254	9550	10964
3	0.2649	2512	10000000
4	0.2807	64565	25704
5	0.266	30903	10000000
6	0.3012	25704	1000
7	0.2134	107152	10000000
8	0.2737	194984	993
9	0.2508	42658	43652
10	0.05326	31623	10000000
11	0.06869	131826	51
12	0.1345	72444	9772
13	0.1785	204174	347
14	0.133	112201	9120
15	0.1146	199526	10000000
16	0.1999	75858	21380
17	0.2112	32359	10000000
18	0.2417	281838	76
19	0.02813	38019	5370318
20	0.04018	56234	10000000
21	0.06016	1148154	91
22	0.06412	109648	10000000
23	0.1721	645654	98
24	0.0462	1047129	346737
25	0.146	851138	115
26	0.1026	331131	588844
27	0.08962	549541	10000000

Table 5. ANOVA for fatigue at hat-friction plate corner

Source	DF	Seq SS	Adj SS	Adj MS	F	P
A	2	1724.93	1724.93	862.46	73.53	0.001
B	2	683.51	683.51	341.75	29.13	0.004
C	2	127.06	127.06	63.53	5.42	0.073
D	2	126.26	126.26	63.13	5.38	0.073
E	2	48.36	48.36	24.18	2.06	0.243
F	2	369.65	369.65	184.82	15.76	0.013
G	2	309.89	309.89	154.94	13.21	0.017
H	2	16.01	16.01	8.01	0.68	0.556
I	2	19.11	19.11	9.56	0.81	0.505
K	2	637.57	637.57	318.78	27.18	0.005
L	2	10.55	10.55	5.27	0.45	0.667
Residual	4	46.92	46.92	11.73		
Total	26	4119.81				

Table 6. ANOVA for fatigue at hub mounting surface corner

Source	D F	Seq SS	Adj SS	Adj MS	F	P
A	2	708.7	708.7	354.3	8.73	0.035
B	2	46.0	46.0	23.0	0.57	0.607
C	2	223.0	223.0	111.5	2.75	0.178
D	2	97.8	97.8	48.9	1.20	0.390
E	2	51.5	51.5	25.7	0.63	0.576
F	2	108.1	108.1	54.1	1.33	0.360
G	2	193.0	193.0	96.5	2.38	0.209
H	2	1547.6	1547.6	773.8	19.06	0.009
I	2	634.2	634.2	317.1	7.81	0.042
J	2	41124.5	41124.5	20562.2	506.58	0.000
K	2	21.8	21.8	10.9	0.27	0.777
Residual	4	162.4	162.4	40.6		
Total	26	44918.5				

Figure 2 shows that fatigue life at the hat friction plate corner improves with increase in the thickness of the inboard friction plate, and reduces with a decrease in effective offset. The axial deflection of the disc shows a trend in which the axial deflection decreases with increase in effective stress and inboard plate thickness (see Figure 4). Comparing this with the fatigue at the hub mounting surface corner as shown in Figure 3 that fatigue life at this point increases with an increasing effective offset, and also increases with inboard plate thickness. The sensitivity plots and the ANOVA indicate that in the design of the vented brake disc the dimension of the inboard plate and the effective offset are both critical to the fatigue life of the vented brake disc. Optimal dimensions for the vented brake disc can be determined using these graphs as well as other requirements such as vehicle design or customer requirements to select the dimensions that would give the best design with respect to the disc axial deflection and the fatigue life at the studied sections of the disc.

Table 7. ANOVA for axial deflection of brake disc

Source	DF	Seq SS	Adj SS	Adj MS	F	P
A	2	592.992	592.99	296.50	97.24	0.000
B	2	140.224	140.22	70.11	22.99	0.006
C	2	56.803	56.80	28.40	9.31	0.031
D	2	38.241	38.24	19.12	6.27	0.058
E	2	4.528	4.53	2.26	0.74	0.532
F	2	15.072	15.07	7.54	2.47	0.200
G	2	5.389	5.39	2.70	0.88	0.481
H	2	37.008	37.01	18.50	6.07	0.061
J	2	13.882	13.88	6.94	2.28	0.219
K	2	48.985	48.99	24.49	8.03	0.040
L	2	13.180	13.18	6.590	2.16	0.231
Residual	4	12.196	12.196	3.05		
Total	26	978.500				

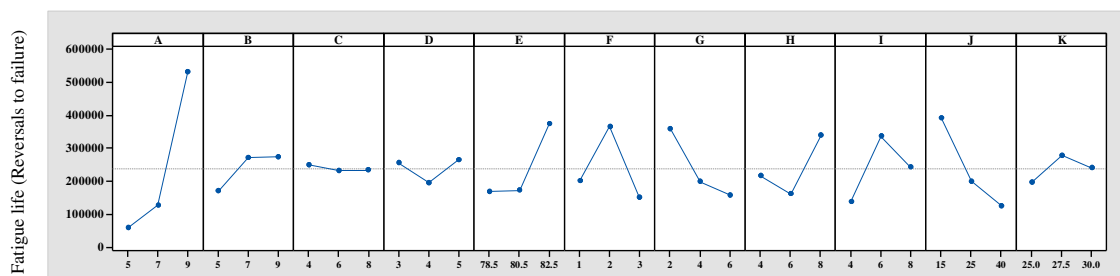


Figure 2. Effect diagram for sensitivity of fatigue at hat-friction plate corner

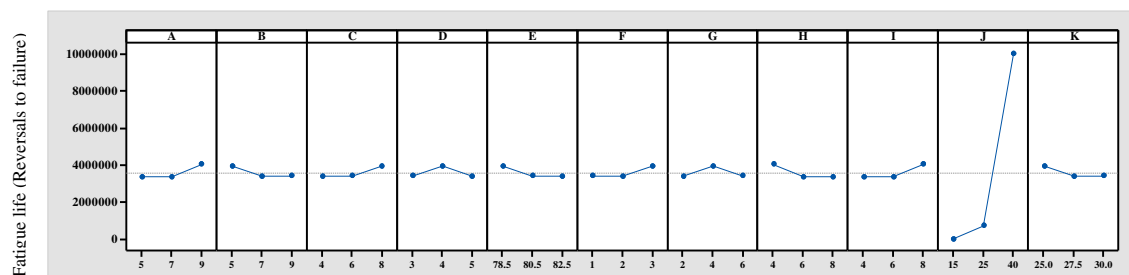


Figure 3. Effect diagram for sensitivity of fatigue at hub mounting surface corner

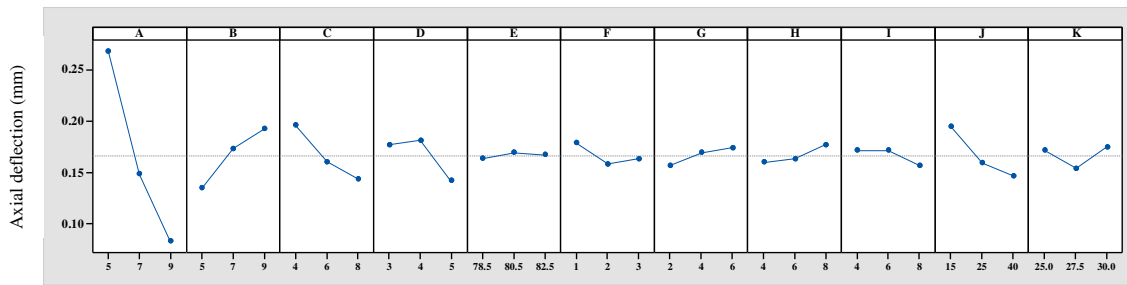


Figure 4. Effect diagram for sensitivity of axial deflection

## 5. Conclusions

This paper presents a CAE/FEA simulation and design of experiment methodology using Taguchi method to study the influence of geometric design factors on the fatigue life at selected critical areas of a vented brake disc and the axial deflection of the vented brake disc due to thermal inputs. The results obtained indicate that two important geometric features that influence these life performance measures are the inboard plate thickness and the length of the effective offset.

In designing of the vented brake disc more design effort should be concentrated on these features so as to obtain the best service life. Future work would be carried out to determine the influence of uncertainties on the fatigue life of the vented brake discs due to the identified significant geometric design features.

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